Fractal-Based Point Processes

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Appendix C
List of Symbols

C.1 ROMAN SYMBOLS

Symbol	Description	Reference
a	Modulation depth	Eq. (13.16)
a–d	General continuous parameters	
a	Shot-noise impulse response function area	Eq. (10.12)
A	Short-time (high-frequency) cutoff	Eq. (7.1)
$A(T)$	Normalized Haar-wavelet variance	Sec. 3.4.3
$A_W(T)$	Normalized general-wavelet variance	Sec. 5.4.3
$A_\tau(k)$	Normalized interval Haar-wavelet variance	Sec. 3.3.4
$A^{(2)}(T)$	Normalized Haar-wavelet covariance	Eq. (3.75)
$\mathcal{A}$	Multiplicative rate constant	Prob. 6.8
$\mathbf{A}_L$	Lorentz vector potential	Prob. 10.6
b	Major axis of planetary elliptical orbit	Sec. 2.7.2
B	Long-time (low-frequency) cutoff	Eq. (7.1)
$B(t)$	Brownian motion	Sec. 2.4.2
$B^*(t)$, $B^\dagger(t)$	Scaled versions of Brownian motion	Sec. 2.4.2
$B_H(t)$	Fractional Brownian motion	Sec. 6.1
$B'_H(t)$	Fractional Gaussian noise	Sec. 6.2.1
$B'_{H2}(t,v)$	Rectangularly filtered fractional Gaussian noise	Eq. (6.11)
$\mathsf{B}_H(t)$	Multifractal version of $B_H(t)$	Sec. 5.5.1
$\mathcal{B}$	Minimal covering width	Sec. 7.2.5
$\mathbf{B}$	Magnetic field vector	Prob. 10.6
c	Speed of light in free space	Prob. 10.6
C_{Euler}	Euler's constant ($\doteq$ 0.5772156649)	Eq. (12.37)
C_n	Cumulant (semi-invariant)	Eq. (3.9)
C_μ	Rate coefficient of variation	Sec. 4.3
C_τ	Interevent-interval coefficient of variation	Eq. (3.5)
$C_{\psi,N}(a,b)$	Continuous-time wavelet transform	Sec. 3.4.3
$\mathcal{C}$, $\mathcal{C}_n$	Cantor set, nth-stage approximation	Sec. 2.4.1
$\mathcal{C}^F$, $\mathcal{C}^F_n$	Fat Cantor set, nth-stage approximation	Sec. 2.4.1
d	Distance	Sec. 2.7.2
$dN(t)$	Point process	Sec. 3.2
D	Fractal dimension	Sec. 3.5.4
D_d	Diffusion-constant exponent	Prob. 10.8
D_q	Generalized (Rényi) dimension	Sec. 3.5.4
D_s	Spectral-dimension exponent	Prob. 10.8
D_{E}	Euclidian dimension	Sec. 2.7.3
D_{HB}	Hausdorff–Besicovitch dimension	Sec. 3.5.4
D_0	Capacity (box-counting) dimension	Sec. 2.1.1
D_1	Information dimension	Sec. 3.5.4
D_2	Correlation dimension	Sec. 3.5.4
$\mathrm{erfc}(\cdot)$	Complementary error function	Eq. (7.16)
E	Energy	Prob. 7.10

Symbol	Description	Reference
$\mathrm{E}[\cdot]$	Expectation or mean	
$\mathbf{E}$	Electric field vector	Prob. 10.6
f	Frequency in cycles per unit time (Hz)	Sec. 3.3.3
$f(\cdot)$	General function	
f_S	Spectrum fractal cutoff frequency	Eq. (5.1)
f	Frequency in cycles per interval (dimensionless)	Sec. 3.3.3
$F(T)$	Normalized variance	Sec. 3.4.2
$\mathcal{F}\{\cdot\}$	Fourier transform	Sec. 9.4
F	Force or field	Sec. 2.7.2
$g(\cdot)$	General function	
$G(t)$	Coincidence rate	Sec. 3.5.1
h	Planck's constant	Prob. 10.6
$h(K, t)$	Shot-noise impulse response function	Sec. 9.1
$h_T(K, t)$	Integrated-shot-noise impulse response function	Eq. (10.2)
H	Hurst exponent	Chapter 6
$H(f)$	Fourier transform of impulse response function	Eq. (9.25)
$\mathbf{H}$	Magnetic induction vector	Prob. 10.6
i	$\sqrt{-1}$	
i	Space-charge-limited current	Sec. 2.7.2
$\mathrm{int}(\cdot)$	Integer function	
j	Quantum number of a simple system	Sec. 2.7.2
J	Čerenkov relativistic factor	Prob. 10.6
k–n	General counting variables	
k_2	Offset delay number for detrended fluctuations	Sec. 3.3.6
K	Shot-noise impulse response amplitude	Sec. 9.1
$\mathrm{K}_\gamma(x)$	Modified Bessel function of the second kind	Eq. (7.5)
l	Wavelet transform position index	Sec. 3.3.4
l_d	Correlation length	Prob. 10.9
ℓ	Decimation parameter	Sec. 11.2.2
$\ln(\cdot)$	Natural logarithm function (base e)	
$\log(\cdot)$	Logarithm function (arbitrary base)	
L	Duration of a data set	Sec. 3.5.2
$L(\cdot)$	Slowly varying function	Eq. (7.24)
$\mathcal{L}(\cdot)$	Lebesgue measure	Sec. 2.4.1
$\max(\cdot)$	Function returning largest argument	
$\min(\cdot)$	Function returning smallest argument	
$\mathrm{mod}(\cdot)$	Modulus function	
m	Order of the gamma renewal process	Prob. 4.7
M	General counting variable	
$M(\epsilon)$	Number of covering elements	Sec. 2.1.1
$\mathsf{M}(t)$	Multifractal process	Sec. 5.5.1
$\mathcal{M}$	Mass of an aggregated particle	Prob. 9.3

Symbol	Description	Reference
n_v	Number of contiguous vanishing moments of $\psi(t)$	Sec. 5.2.5
n	Refractive index	Prob. 10.6
$N(t)$	Counting process	Sec. 3.2
$N(L)$	Number of events from origin to time L	Sec. 3.5.4
$N_a(t)$	Arrival counting process at a queue	Sec. 13.1.1
$N_s(t)$	Service counting process at a queue	Sec. 13.1.1
$\widetilde{N}(f)$	Point-process Fourier transform	Eq. (B.254)
$\mathcal{N}(0,1)$	Normalized Gaussian random variable	Prob. 6.5
$p_{\cdot}(\cdot)$	Probability density function	Eq. (3.3)
$p_Q(n,t)$	Queue-length distribution	Sec. 13.1.1
$p_Z(n;T)$	Counting distribution	Eq. (3.28)
$p_\infty(n,t)$	Queue-length distribution (infinite buffer size)	Eq. (13.4)
$P_{\cdot}(\cdot)$	Probability distribution function	Eq. (3.3)
$\Pr\{\cdot\}$	Probability	
P_B	Buffer overflow (blocking) probability	Sec. 13.1.1
q	Generalized-dimension index	Sec. 3.5.4
$q(E)$	Trap waiting time	Prob. 7.10
q	Charge constant	Prob. 6.8
$Q(t)$	Queue length	Sec. 13.1.1
Q_m	Maximum queue length (buffer size)	Sec. 13.1.1
r	Bernoulli-trial success probability	Sec. 8.1.1
$r(t)$	Deletion recovery function	Chapter 11
r	Distance or deformation	Sec. 2.7.2
$\mathrm{Re}\{\cdot\}$	Real part	
$R_Z(k,T)$	Count autocorrelation	Sec. 3.4.4
$R_\tau(k)$	Interval autocorrelation	Sec. 3.3.2
$R_2(k)$	Normalized count autocovariance	Sec. 12.3.3
$\mathcal{R}$	Thermodynamic gas constant	Prob. 6.8
s	Absolute time	Sec. 3.2
s	Measurement scale	Eq. (1.1)
$\mathrm{sgn}(x)$	Sign of x	
$S_N(f)$	Point-process spectrum	Sec. 3.4.5
$S_{W,X}(t,f)$	Wigner–Ville spectrum	Eq. (6.9)
$S_Z(f,T)$	Count spectrum	Sec. 3.5.2
$S_\lambda(f,T)$	Rate spectrum	Sec. 3.4.5
$S_\tau(\mathsf{f})$	Interval spectrum	Sec. 3.3.3
$S_N^{(2)}(f)$	Point-process cross-spectrum	Eq. (3.76)
$S_\lambda^{(2)}(f,T)$	Rate cross-spectrum	Eq. (3.77)
$\mathsf{S}_\tau(t)$	Interval survivor function	Sec. 3.3.1
$\mathsf{S}_\vartheta(t)$	Recurrence-time survivor function	Prob. 11.7
$\mathcal{S}$	Linear-fit sum	Eq. (A.4)
$\mathbf{S}$	Poynting vector	Prob. 10.6

Symbol	Description	Reference
t	Absolute time	Sec. 3.2
t_G	Coincidence-rate fractal cutoff time	Eq. (5.12)
T	Counting duration or counting window	Sec. 3.2
T_A	Normalized-Haar-wavelet-variance fractal cutoff time	Eq. (5.2)
T_{Dq}	Transition time for $\eta_q(T)$	Prob. 5.5.3
T_F	Normalized-variance fractal cutoff time	Eq. (5.11)
T_R	Autocorrelation fractal cutoff time	Eq. (5.14)
$\mathcal{T}$	Absolute temperature	Prob. 6.8
T	Planetary orbital period	Sec. 2.7.2
u–z	General continuous variables	
$u(\mathbf{x}, t)$	Particle concentration at position $\mathbf{x}$ and time t	Prob. 10.8
u_0	Initial particle concentration	Prob. 10.8
$U(k)$	Rescaled range analysis (R/S)	Sec. 3.3.5
$U_2(k)$	Normalized rescaled-range statistic	Sec. 12.3.4
v	Separation time	Sec. 6.2.1
v	Scalar velocity	Prob. 10.6
$\mathbf{v}$	Vector velocity	Prob. 10.6
V	Voltage	Prob. 6.8
$\mathrm{Var}[\,\cdot\,]$	Variance	
$W(\mathcal{B})$	Expected time between coverings	Eq. (7.20)
$W_{n,k}$	Multiplicative-process weighting factors	Sec. 5.5.1
$W_{\psi,\tau}(k, l)$	Discrete-time wavelet transform	Sec. 3.3.4
$\mathbf{x}$	Position vector	Prob. 10.8
$X(t)$	Continuous-time process	Sec. 6.1.2
$X_T(t)$	Integrated shot-noise process	Eq. (10.2)
$X_\Sigma(t)$	Binomial-noise process	Eq. (8.14)
$Y(k)$	Detrended fluctuation analysis	Sec. 3.3.6
$Y_2(k)$	Normalized detrended-fluctuation statistic	Sec. 12.3.5
z	Exponent in fractal Bartlett–Lewis process	Sec. 10.6.4
$Z_k(T)$	Event count	Sec. 3.2

C.2 GREEK SYMBOLS

Symbol	Description	Reference
α_x	Fractal exponent obtained from statistic x	Sec. 5.1.6
β	Fractal-shot-noise exponent	Sec. 9.1
γ	Fractal-renewal-process exponent	Chapter 7
$\Gamma(x)$	Complete Eulerian gamma function	Eq. (4.44)
$\Gamma(x,\, a)$	Incomplete Eulerian gamma function	Eq. (9.6)
$\delta(t)$	Dirac delta function	Sec. 3.5.1
Δ	Diffusion constant	Prob. 10.8
ϵ	Small number	Sec. 2.1.1
ε	Small number parameter	Sec. 12.3.9
ζ	Stable-distribution parameter	Sec. 9.2
$\eta_q(T)$	Generalized-dimension scaling function	Sec. 3.5.4
$\eta_0(T)$	Capacity-dimension scaling function	Sec. 3.5.4
θ	Phase angle	Prob. 4.10
$\vartheta(t)$	Forward recurrence time	Eq. (3.10)
κ	Boltzmann's constant	Prob. 7.10
$\lambda_k(t)$	Sample rate (measured value)	Eq. (3.27)
$\Lambda(t)$	Integrated rate (model property)	Eq. (4.28)
μ	Fixed rate of a point process (model property)	Sec. 4.1
$\mathrm{E}[\mu]$	Expected rate of a point process (model property)	Sec. 3.5.1
$\mu(t)$	Varying rate of a point process (model property)	Sec. 3.5.1
μ_a	Arrival rate at a queue	Sec. 13.1.1
μ_s	Service rate at a queue	Sec. 13.1.1
ν	Frequency of electromagnetic radiation	Prob. 10.6
ξ	Weibull distribution parameter	Sec. 13.3.3
$\rho(s,t)$	Normalized autocorrelation	Eq. (6.28)
ρ_μ	Service ratio (server utilization)	Eq. (13.3)
$\varrho_\tau(k)$	Interval serial correlation coefficient	Eq. (3.17)
σ	Standard deviation	Eq. (3.4)
τ	Interevent interval	Sec. 3.2
τ_e	Effective dead time	Eq. (11.21)
τ_f	Fixed dead time	Eq. (11.16)
τ_w	Waiting time in a queue	Eq. (13.6)
ϕ_L	Lorentz scalar potential	Prob. 10.6
$\phi_\tau(\omega)$	Characteristic function	Eq. (3.6)
$\varphi(f)$	Fourier transform of wavelet $\psi(x)$	Eq. (5.41)
Φ	Smaller of counting and impulse-response times	Eq. (A.125)
χ^2	Linear-fit error	Eq. (A.3)
$\psi(x)$	Mother wavelet (time domain)	Sec. 3.3.4
Ψ	Integrate-and-reset threshold	Fig. 4.1
ω	Angular frequency (radians per unit time)	Eq. (4.46)
Ω	Double integral in normalized variance	Eq. (A.136)

C.3 MATHEMATICAL SYMBOLS

Symbol	Description	Reference
$*$	Complex conjugation	Sec. 3.5.5
$\star$	Convolution	Sec. 4.2
$\otimes$	Vector cross product	Prob. 10.6
$\widehat{x}$	Estimate of x	
!	Factorial of preceding expression	
$\equiv$	Definition	
$\doteq$	Very close to	
$\approx$	Approximately equal to	
$\sim$	Varies as	
$\ll$	Much less than	
$\gg$	Much greater than	
$0-$	Number infinitesimally smaller than zero	
$0+$	Number infinitesimally greater than zero	